Complete structure dependent analysis of the decay $P \to l^+l^-$

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(Dated: March 25, 2009)

Abstract

We use the Mellin-Barnes representation in order to improve the theoretical estimate of mass corrections to the width of light pseudoscalar meson decays into a lepton pair, $P \to l^+l^-$. The full resummation of the terms $\left(M^2/\Lambda^2\right)^n$, $\left(m^2/M^2\right)^n$and $\left(m^2/\Lambda^2\right)^n$ to the decay amplitude is performed, where m is the lepton mass, M is the meson mass and $\Lambda \approx m_\rho$ is the characteristic scale of the $P \to \gamma^*\gamma^*$ form factor. The total effect of mass corrections is quite important for $\eta(\eta')$ decays. We also comment on the estimation of the hadronic light-by-light scattering contribution to the muon anomalous magnetic moment in the chiral perturbation theory.

PACS numbers:

I. INTRODUCTION

The theoretical study of the π^0 and $\eta\,(\eta')$ mesons decaying into lepton pairs and the comparison with the experimental rates offers an important low-energy test of the standard model. The situation with these decays became more pressing after recent KTeV E799-II experiment at Fermilab [1] in which the pion decay into an electron-positron pair was measured with high accuracy using the $K_L \to 3\pi$ process as a source of tagged neutral pions

$$B^{\mathrm{KTeV}}_{\mathrm{no-rad}}\left(\pi^0 \to e^+e^-\right) = (7.49 \pm 0.29 \pm 0.25) \cdot 10^{-8}. \quad (1)$$

The theory for the decay of pseudoscalar mesons to a lepton pair is known for decades [2, 3, 4, 5]. The main limitation for realistic prediction of these processes comes from the large distance contributions of the strong sector of the standard model where the perturbative QCD theory does not work. However, it was shown in [6] that theoretical uncertainty can be significantly reduced by using CELLO and CLEO data [7, 8] and QCD constraints on the transition form factors $P \to \gamma^*\gamma^*$. As a result, the standard model prediction gives [6]

$$B^{\mathrm{Theor}}\left(\pi^0 \to e^+e^-\right) = (6.2 \pm 0.1) \cdot 10^{-8}, \quad (2)$$

which is 3.3σ below the KTeV result (1). It is extremely important to trace possible sources of the discrepancy between the experiment and theory. There are a number of possibilities: (i) problems with (statistic) experiment procession, (ii) inclusion of QED radiation corrections by KTeV is incomplete, (iii) unaccounted mass corrections are important, and (iv) effects of new physics. At the moment, the last possibility was reinvestigated.

In [9], the contribution of QED radiative corrections to the $\pi^0 \to e^+e^-$ decay, which must be taken into account when comparing the theoretical prediction (2) with the experimental data, was revised. In comparison with earlier studies [10] the main progress made in [9] consists in detailed analysis of the $\gamma^*\gamma^* \to e^+e^-$ subprocess and revealing the dynamics of long and short distances. Occasionally, the final result agrees well with earlier prediction based on calculations [10] and, thus, the KTeV analysis of radiative corrections is confirmed.

There are quite few attempts in the literature to explain the excess of the experimental data on the $\pi^0 \to e^+e^-$ decay over the standard model predictions as a manifestation of physics beyond the Standard Model. In Ref. [11], it was shown that this excess could be explained within the currently popular model of light dark matter involving a low mass (~ 10

MeV) vector bosons U_μ which presumably couples to the axial-vector currents of quarks and leptons. Another possibility was proposed in Ref. [12, 13] in interpreting the same experimental effect as the contribution of the light CP-odd Higgs boson appearing in the next-to-minimal supersymmetric Standard Model. However, there are other extensions of the Standard Model deserving studies in this context. In particular, supersymmetric models with R-parity violation and leptoquark models suggest contributions to this process which under certain circumstances may be significant. Specific examples of such contributions are related to the exchange by the t-squark and the SU_{2L} singlet leptoquarks with the couplings to u,d-quarks and electrons not stringently constrained from other known processes [14, 15].

In the present paper, we focus on the mass corrections to the width of light pseudoscalar meson decays into a lepton pair, $P \to l^+l^-$. We show that these corrections are under theoretical control and do not help resolving the problem of disagreement of the KTeV data on the $\pi^0 \to e^+e^-$ decay with the standard model prediction. However, the mass corrections are quite important for realistic predictions for $\eta\,(\eta')$ decays to a lepton pair.

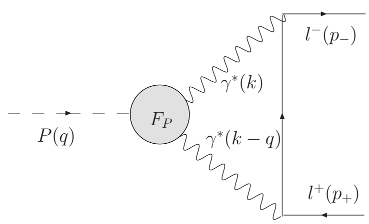

FIG. 1: Triangle diagram for the $P \to l^+l^-$ process with the pseudoscalar meson form factor $P \to \gamma^*\gamma^*$ in the vertex.

In the lowest order of QED perturbation theory, the decay of the neutral meson, $P(q) \to l^-(p_-) + l^+(p_+)$, $\quad q^2 = M^2, \quad p_\pm^2 = m^2$, ($M$ meson mass, m lepton mass) is described by the one-loop Feynman amplitude (Fig. 1) corresponding to the conversion of the neutral meson through two virtual photons into a lepton pair. The normalized branching ratio is given by [2, 3, 4, 5]

$$R_0(P \to l^+l^-) = \frac{B_0\,(P \to l^+l^-)}{B\,(P \to \gamma\gamma)} = 2\beta\left(M^2\right)\left(\frac{\alpha m}{\pi M}\right)^2 |\mathcal{A}\left(M^2\right)|^2, \tag{3}$$

where $\beta\left(q^2\right) = \sqrt{1 - 4m^2/q^2}$ and the reduced amplitude is

$$\mathcal{A}\left(q^2\right) = \frac{2}{q^2}\int \frac{d^4k}{i\pi^2}\frac{(qk)^2 - q^2k^2}{(k^2 + i\epsilon)\,[(q-k)^2 + i\epsilon]\,[(p_- - k)^2 - m^2 + i\epsilon]} F_{P\gamma^*\gamma^*}(-k^2, -(q-k)^2), \tag{4}$$

with the transition form factor $F_{P\gamma^*\gamma^*}(-k^2,-q^2)$ being normalized as $F_{P\gamma^*\gamma^*}(0,0)=1$. It is convenient to introduce the mass ratio parameters:

$$x=(m/\Lambda)^2\,,\quad u=(m/M)^2\,,\quad z=x/u=(M/\Lambda)^2\,, \tag{5}$$

where $\Lambda\sim M_\rho$ is a parameter characterizing the transition form factor. In the physically interesting cases the parameter x is very small: $(m_e/\Lambda)^2\lesssim 10^{-7},(m_\mu/\Lambda)^2\lesssim 10^{-2}$. At the same time, we consider the amplitude in the leading order in the fine coupling constant $\alpha\sim 10^{-2}$. Thus, in the following it is reasonable to neglect the x power dependence of the amplitude keeping only the dependence on the u and z variables [1]. The aim of this work is to improve the previous calculations of the amplitude $\mathcal{A}\left(M^2\right)$ of the $P\to l^+l^-$ decay by taking into account all order mass corrections.

II. RESUMMATION OF POWER CORRECTIONS TO THE AMPLITUDE

We evaluate the amplitude $\mathcal{A}\left(q^2\right)$ following the way used in [5, 6, 16]. Let us transform the integral in (4) to the Euclidean metric $k_0\to ik_4$. The corresponding integral is convergent due to decreasing of $F_{P\gamma^*\gamma^*}(k^2,(q-k)^2)$ in the Euclidean region. It is convenient to introduce the modified Mellin-Barnes transformation for the meson form factor

$$F_{P\gamma^*\gamma^*}(k^2,(q-k)^2)=\frac{1}{(2\pi i)^2}\int_{\sigma+iR^2}dz_1dz_2\Phi\left(z_1,z_2\right)\Gamma\left(z_1\right)\Gamma\left(z_2\right)\left(\frac{\Lambda^2}{k^2}\right)^{z_1}\left(\frac{\Lambda^2}{\left(k-q\right)^2}\right)^{z_2}, \tag{6}$$

where Λ is the characteristic scale for the form factor, the vector $\sigma=(\sigma_1,\sigma_2)\in\mathbb{R}^2$, and $\Phi\left(z_1,z_2\right)$ is the inverse Mellin-Barnes transform of the form factor

$$\Phi\left(z_1,z_2\right)=\frac{1}{\Gamma\left(z_1\right)\Gamma\left(z_2\right)}\int_0^\infty dt_1\int_0^\infty dt_2t_1^{z_1-1}t_2^{z_2-1}F_{P\gamma^*\gamma^*}\left(t_1,t_2\right) \tag{7}$$

which is an analytical function of z_1 and z_2. Introducing Feynman parameterization in (4) we convert the $k-$loop integral into the integrals in Feynman parameters which can be expressed in terms of Γ-functions. Then we obtain the following Mellin-Barnes representation for the amplitude [16]

$$\mathcal{A}\left(q^2\right)=-\int_{\sigma+i\mathbb{R}^3}\frac{dz_1dz_2dz_3}{\left(2\pi i\right)^3}x^{-z_1-z_2}u^{-z_3}\Phi\left(z_1,z_2\right)\left[3-\left(2+\frac{1}{2u}\right)\left(z_1+z_2+z_3\right)\right] \tag{8}$$
$$\frac{\Gamma\left(-z_3\right)\Gamma\left(1+z_1+z_3\right)\Gamma\left(1+z_2+z_3\right)\Gamma\left(z_1+z_2+z_3\right)\Gamma\left(1-2\left(z_1+z_2+z_3\right)\right)}{z_1z_2\Gamma\left(3-z_1-z_2\right)},$$

[1] Partially, the dependence of the amplitude $\mathcal{A}\left(q^2\right)$ on the x parameter was taken into account in [16].

with σ in the 3-dimensional region in space of real parts of z_i chosen so that the integration path $\sigma + i\mathbb{R}^3$ does not intersect the Γ-function singularities. Finally, we are able to expand the integral over the x, u and z mass ratios by closing the contours of integration in the appropriate manner and summing up the obtained series. Neglecting the dependence of the amplitude $\mathcal{A}\left(q^2\right)$ on the powers of small x parameter (but keeping $\ln x$) we arrive at the following representation:

$$\begin{aligned}\mathcal{A}\left(M^2\right) = &\frac{1}{\beta\left(u\right)}\left[\frac{1}{4}\ln^2\left(y\left(u\right)\right) + \frac{\pi^2}{12} + \mathrm{Li}_2\left(-y\left(u\right)\right)\right]\\ &+ i\frac{\pi}{2\beta\left(u\right)}\ln\left(y\left(u\right)\right)\\ &+ \frac{3}{2}\ln\left(x\right) - \frac{5}{4} + \frac{3}{2}\int_0^\infty dt \ln\left(\frac{t}{\Lambda^2}\right) F^{(1,0)}_{P\gamma^*\gamma^*}\left(t,t\right)\\ &+ A_z,\end{aligned} \tag{9}$$

where

$$y\left(q^2\right) = \frac{1-\beta\left(q^2\right)}{1+\beta\left(q^2\right)},$$

and the correction to $\mathcal{A}\left(M^2\right)$ is expressed as

$$\begin{aligned}A_z = &\left(\ln x + \frac{3}{2}\right)\int_0^1 \frac{ds}{s}\left(1-s\right)^2\left[1 - F_{P\gamma^*\gamma^*}\left(-sz,0\right)\right] - \frac{z}{3}F^{(1,0)}_{P\gamma^*\gamma^*}\left(0,0\right)\\ &- \frac{z}{6}\int_0^\infty \frac{dt}{t^2}\left[F_{P\gamma^*\gamma^*}(t,t) - F_{P\gamma^*\gamma^*}(0,t) - tF^{(1,0)}_{P\gamma^*\gamma^*}\left(0,t\right)\right]\\ &+ \int_0^1 \frac{ds}{s}\left(1-s\right)^2\int_0^1 \frac{dy}{\left(1-y\right)y}\\ &\times\left[1 - 2y + y^3 + \frac{3}{2}yF_{P\gamma^*\gamma^*}\left(-sz,0\right) - \left(1 - \frac{1}{2}y + y^3\right)F_{P\gamma^*\gamma^*}\left(-syz,0\right)\right]\\ &+ \frac{1}{2}\int_0^1 \frac{ds}{s}\left(1-s\right)^2\int_0^\infty dt\ln(t)\left[2F^{(1,0)}_{P\gamma^*\gamma^*}\left(t,0\right) - F^{(1,0)}_{P\gamma^*\gamma^*}\left(t-sz,0\right) - F^{(1,0)}_{P\gamma^*\gamma^*}\left(t,-sz\right)\right]\\ &- \frac{3z}{\pi}\int_0^1 ds\left(1-s\right)^2\int_0^{\pi/2} dv\cos v\int_0^\infty dt\frac{1}{\left(stz\right)^{1/2}}\,\mathrm{Im}\,F^{(1,0)}_{P\gamma^*\gamma^*}\left(t - zs + 2i\left(stz\right)^{1/2}\cos v, t\right).\end{aligned} \tag{10}$$

In the above expressions $F^{(\alpha,\beta)}_{P\gamma^*\gamma^*}(s,t)$ denotes the derivatives of an order of α and β in the corresponding arguments of the form factor.

The first and second lines in (9) are the structure independent parts of the amplitude. These expressions agree with obtained earlier results. In particular, they agree with the results of the dispersion approach of [17] and the chiral perturbation theory [18, 19]. The derivation of the amplitude by using the dispersion approach tacitly assumes that the imaginary part of the off-shell amplitude $\mathcal{A}(q^2)$ is the second line of (9) with M^2 substituted

by q^2. Thus, it takes into account only the structure independent part (dependence on the parameter u) and is insensitive to the details of the transition form factor. The same result appears in the framework of the leading order of the chiral perturbation theory [18, 19], because at this order one also does not take into account the form factor. The integral in the second line of (9) was estimated in [6] by using CELLO and CLEO data on the meson transition form factors and constraints following from operator product expansion (OPE) in QCD. Our new result concerns the structure dependent part of the amplitude A_z as the combination of the integrals of the meson transition form factor over both the spacelike and timelike regions.

If we retain only the terms linear in z we get

$$A_z^{(1)} = -\frac{z}{6}\left\{\left(-2\ln x + \frac{14}{3}\right) F^{(1,0)}_{P\gamma^*\gamma^*}(0,0) + 3\int_0^\infty dt F^{(1,1)}_{P\gamma^*\gamma^*}(t,t) \right. \\ - \int_0^\infty dt \ln(t) \left[F^{(1,1)}_{P\gamma^*\gamma^*}(t,0) + F^{(2,0)}_{P\gamma^*\gamma^*}(t,0)\right] \\ \left. + \int_0^\infty \frac{dt}{t^2}\left[F_{P\gamma^*\gamma^*}(t,t) - F_{P\gamma^*\gamma^*}(0,t) - tF^{(1,0)}_{P\gamma^*\gamma^*}(0,t)\right]\right\}. \tag{11}$$

In order to estimate the correction A_z to the amplitude related to the meson mass, we consider the simplest parameterization of the transition form factor given by the naive vector meson dominance model

$$F^{\mathrm{VMD}}_{P\gamma^*\gamma^*}(s,t) = \frac{M_V^4}{(M_V^2+s)(M_V^2+t)}. \tag{12}$$

In that case, $z = (M/M_V)^2$ and the correction is

$$A_z^{\mathrm{VMD}} = \ln(x)\left(\frac{(1-z)^2}{z^2}\ln(1-z) - \frac{3}{2} + \frac{1}{z}\right) - \frac{2z-1}{z^2}\mathrm{Li}_2(z) + \\ + \frac{(1-z)}{2z^2}\ln(1-z)\left[(1-z)\ln(1-z) + 6z - 4\right] + \frac{z}{6} + \frac{3}{2} - \\ - \frac{3}{4z^2}\left\{-\mathrm{arctg}^2\left[\frac{\sqrt{4z-z^2}(2-z)}{2-4z+z^2}\right] \right. \\ \left. + \sqrt{\frac{z}{4-z}}\left(8-6z+z^2\right)\mathrm{arctg}\left[\frac{\sqrt{4z-z^2}(2-z)}{2-4z+z^2}\right]\right\} \tag{13}$$

which in the linear approximation ($z << 1$) reduces to

$$A_z^{(1),\mathrm{VMD}} = \frac{z}{6}\left(-2\ln x + \frac{5}{3}\right). \tag{14}$$

Taking $M_V = 770$ MeV the resulting branchings are given in the Table and compared with existing experimental data. The so-called unitary bound appears if in (3) only the

TABLE I: Values of the branchings $B\left(P \to l^+l^-\right)$ obtained in our approach and compared with the available experimental results.

R_0	Unitary bound	CLEO bound	CLEO+OPE	This work	Experiment
$R_0\left(\pi^0 \to e^+e^-\right) \times 10^8$	≥ 4.69	$\geq 5.85 \pm 0.03$	6.23 ± 0.12	6.26	7.49 ± 0.38 [1]
$R_0\left(\eta \to \mu^+\mu^-\right) \times 10^6$	≥ 4.36	$\leq 6.23 \pm 0.12$	5.12 ± 0.27	4.64	5.8 ± 0.8 [20, 21]
$R_0\left(\eta \to e^+e^-\right) \times 10^9$	≥ 1.78	$\geq 4.33 \pm 0.02$	4.60 ± 0.09	5.24	$\leq 2.7 \cdot 10^4$ [22]
$R_0\left(\eta' \to \mu^+\mu^-\right) \times 10^7$	≥ 1.35	$\leq 1.44 \pm 0.01$	1.364 ± 0.010	1.30	
$R_0\left(\eta' \to e^+e^-\right) \times 10^{10}$	≥ 0.36	$\geq 1.121 \pm 0.004$	1.178 ± 0.014	1.86	

imaginary part of the amplitude which is model independent is taken into account. The CLEO bound corresponds to the estimate of the real part of the amplitude basing on the CELLO and CLEO data on the meson transition from factors. The fourth column of the Table contains the predictions where in addition the constraint from OPE QCD on the transition form factor for arbitrary photon virtualities is taken into account[6].

As expected, a visible change occurs only for $\eta\,(\eta')$ meson decays. It is interesting that for η decay to muons the mass correction shifts the theoretical prediction in the direction to the unitary bound and thus opposite to the experimental result [21]. This is because the real part of the amplitude for this process taken at the physical point $q^2 = M_\eta^2$ remains negative and a positive shift due to mass correction reduces the absolute value of the real part of the amplitude $\left|\mathrm{Re}\,\mathcal{A}\left(M_\eta^2\right)\right|$. In this situation, new measurements of $\eta \to \mu^+\mu^-$ would be very desirable.

For η' decays there appear new thresholds in addition to the two-photon one. In general, this violates the so-called unitary bound because the correction A_z gains an imaginary part at $z > 1$

$$\Delta\,\mathrm{Im}\,\mathcal{A} = -\frac{\pi}{\beta}\left(1 - \frac{1}{z}\right)^2 \ln\left(\frac{1+\beta}{1-\beta}\right)\Theta(z-1). \tag{15}$$

As it is seen from the table, this happens for the $\eta' \to \mu\mu$ channel. Nevertheless, it turns out that the predictions for this channel are quite accurate. We checked that more sophisticated models for the transition form factor (the generalized VMD [23] or the effect of meson mixing [24]) do not significantly change the results of the Table.

III. HADRONIC LIGHT-BY-LIGHT CONTRIBUTION TO MUON G-2 IN CHIRAL PERTURBATION THEORY

In [25], the hadronic light-by-light contribution to muon $g-2$, which is enhanced by large logarithms (LL) and a factor of N_c $\left(\sim \mathcal{O}\left(\alpha^3 N_c m_\mu^2\right) LL\right)$, was estimated in the chiral perturbation theory as

$$a_\mu^{\mathrm{LbL,hadr}} = a_{\mu,\mathrm{LL}}^{\mathrm{LbL,hadr}} + a_{\mu,\mathrm{pionloop}}^{\mathrm{LbL,hadr}} + \widetilde{C}, \tag{16}$$

where

$$a_{\mu,\mathrm{LL}}^{\mathrm{LbL,hadr}} = \frac{3}{16}\left(\frac{\alpha}{\pi}\right)^3 \left(\frac{m_\mu}{F_\pi}\right)^2 \left(\frac{N_c}{3\pi}\right)^2 \left\{\frac{1}{4}\ln^2\left(\frac{m_\mu^2}{\Lambda^2}\right) - \frac{1}{2}\ln\left(\frac{m_\mu^2}{\Lambda^2}\right)\left[-f\left(\frac{m_\mu^2}{m_\pi^2}\right) + \frac{1}{2} + \frac{\chi(\Lambda)}{6}\right]\right\},$$

$$f(y) = \frac{1}{6}y^2 \ln y - \frac{1}{6}(2y+13) + \frac{1}{3}(2+y)\sqrt{y(4-y)}\arccos\left(\frac{\sqrt{y}}{2}\right),$$

the constant $\widetilde{C}$ absorbs subdominant structure dependent contributions. The low energy constant $\chi(\Lambda)$ is related to the amplitude for pion decay into an electron-positron pair by[2]

$$\begin{aligned} \chi(\Lambda) &= 2\left(3\ln\left(\frac{m_e^2}{\Lambda^2}\right) - 2\mathcal{A}(0) - 7\right) \\ &= -9 - 6\int_0^\infty dt \ln\left(\frac{t}{\Lambda^2}\right) F_{P\gamma^*\gamma^*}^{(1,0)}(t,t). \end{aligned} \tag{17}$$

Taking $\Lambda = 1$ GeV we obtain for the logarithmic enhanced term

$$a_{\mu,\mathrm{LL}}^{\mathrm{LbL,hadr}} = (3.4 \pm 2.0) \times 10^{-10}, \tag{18}$$

where uncertainty arises from determination of the integral in (17). The previous estimate was $a_{\mu,\mathrm{LL}}^{\mathrm{LbL,hadr}} = \left(5.7^{+5.0}_{-6.0}\right) \times 10^{-10}$[25]. Better accuracy in (18) is due to higher quality of determination of $\chi(1\mathrm{GeV}) = -17.35 \pm 1.20$ by using CELLO and CLEO data for the transition from factor $F_{P\gamma^*\gamma^*}(t,t)$ instead of using the decay $\eta \to \mu^+\mu^-$, as was done in [25] with the result $\chi(1\mathrm{GeV}) = -14^{+4}_{-5}$.

The second term in (16) $a_{\mu,\mathrm{pionloop}}^{\mathrm{LbL,hadr}}$ being of the order $O\left(\alpha^3 N_c^0\right)$ arises from the three-loop graphs with a charged pion loop. It was computed in [27, 28] $a_{\mu,\mathrm{pionloop}}^{\mathrm{LbL,hadr}} = -4.46 \times 10^{-10}$.

[2] The logarithmically enhanced hadronic light-by-light contributions to a_μ are renormalization scheme independent. However, the values $\chi(\Lambda)$, $f(y)$ and the constant $(1/2)$ appearing in the renormalization group equation depend on the choice of a scheme. Thus, the constant $\chi(\Lambda)$ defined in [25] is related to the corresponding constant $\chi^*(m_\rho)$ in [26] by relation $\chi(\Lambda) = -4\left[\chi^*(m_\rho) - 3ln\left(\frac{m_\rho}{\Lambda}\right) + 1\right]$.

Thus, there are large cancellations between $\ln^2$ and ln terms in $a_{\mu\partial,\log}^{\text{LbL,hadr}}$ as well as between $a_{\mu,\log}^{\text{LbL,hadr}}$ and $a_{\mu,\text{pionloop}}^{\text{LbL,hadr}}$ with final result

$$a_{\mu,\chi PT}^{\text{LbL,hadr}} = \left(-1.06 \pm 2.0 + 3.1\widetilde{C}\right) \times 10^{-10}. \tag{19}$$

The conclusion is that in the case of $P \to ll$ decays the large logarithmic terms are dominant and the structure dependent part of the amplitude is a small correction. That is why the predictions for the $P \to ll$ decay branchings are rather stable. This is not the case for the hadronic light-by-light contribution to muon $g-2$. Due to cancellations, the nonleading structure dependent terms become crucially important and the chiral perturbation theory does not provide realistic estimate. It is necessary to use specific models to provide correct predictions for $a_\mu^{\text{LbL,hadr}}$ (see for review [29, 30, 31, 32]).

IV. CONCLUSIONS

Our main conclusion is that the inclusion of radiative and mass corrections is unable to reduce the discrepancy between the standard model prediction for the $\pi^0 \to e^+e^-$ decay rate (2) and experimental result (1). Mass corrections are important to get a reliable prediction for branchings of η-meson to a lepton pair. Moreover, they are small enough to be sensitive to details of the transition form factor. For η' decay modes not only mass corrections but also threshold behavior of the transition form factor are important. Thus, theoretical error for η' decays is less under control comparing with π^0 and η meson decays. Further independent experiments at KLOE [33], WASAatCOSY [34], BES III [35] and other facilities will be crucial for resolution of the problem.

V. ACKNOWLEDGMENTS

A.E.D. acknowledges partial support from the Scientific School grant 4476.2006.2. This work is partially supported by the PBCT project ACT-028 Center of Subatomic Physics.

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